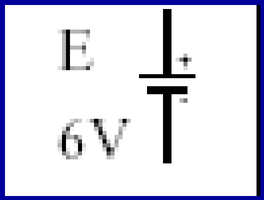
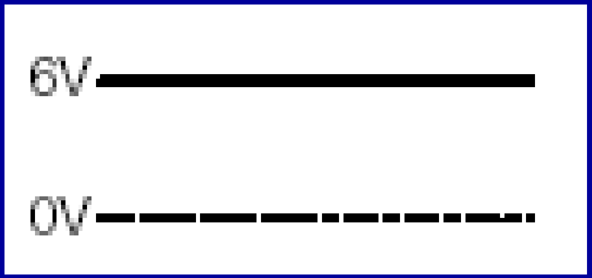
Lecture 3

- AC Fundamentals
- Resonance

Fixed Sources





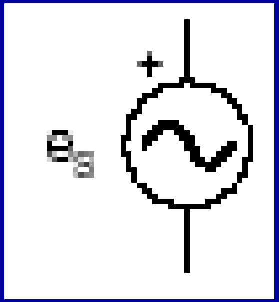
DC

Direct Current

Fixed current or voltage

Value does not vary with time

Alternating Waveforms



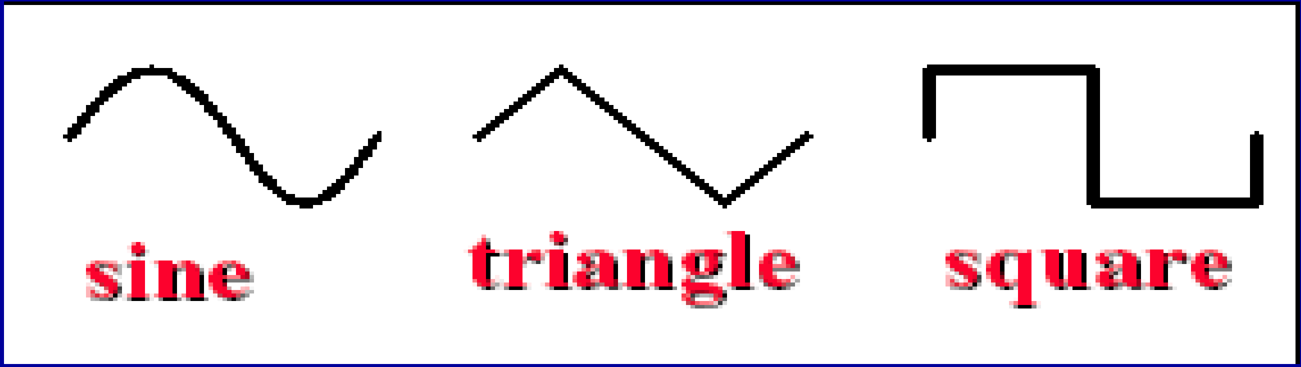
AC

Alternating Current

Alternating current or voltage

Varies with time

Alternating Waveforms



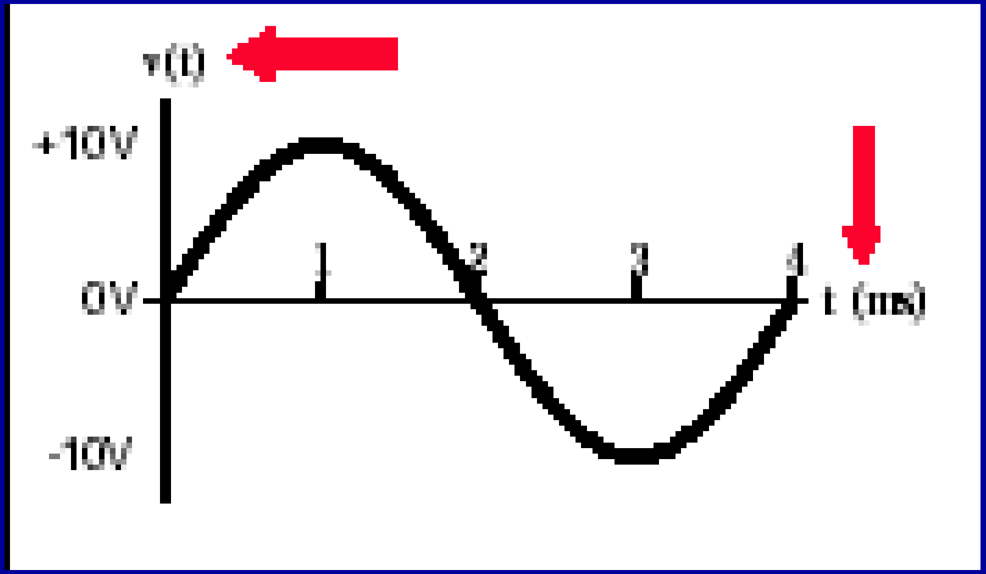
Basic Common Waveforms

Key Concepts – *value varies with time*

Voltage

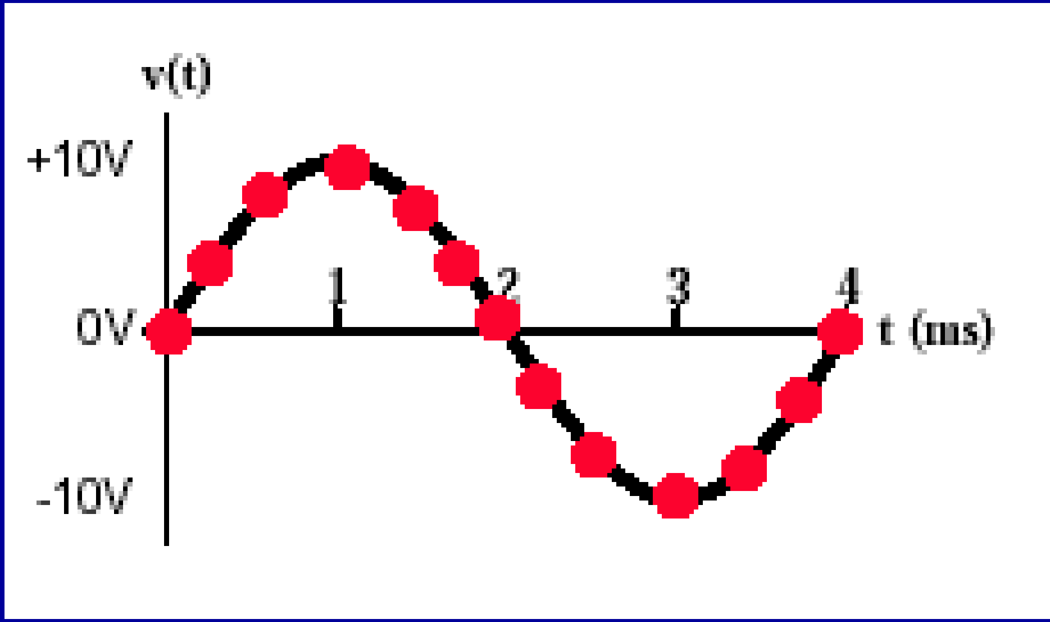
or

Current



time

AC Wave – *value varies with time*

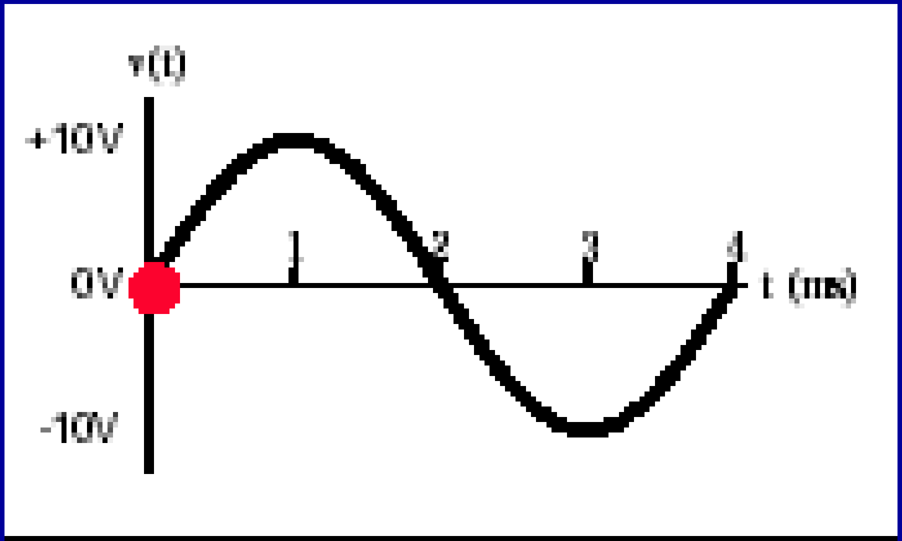


AC Wave – *value varies with time*

0 Volts

at

***t* = 0**

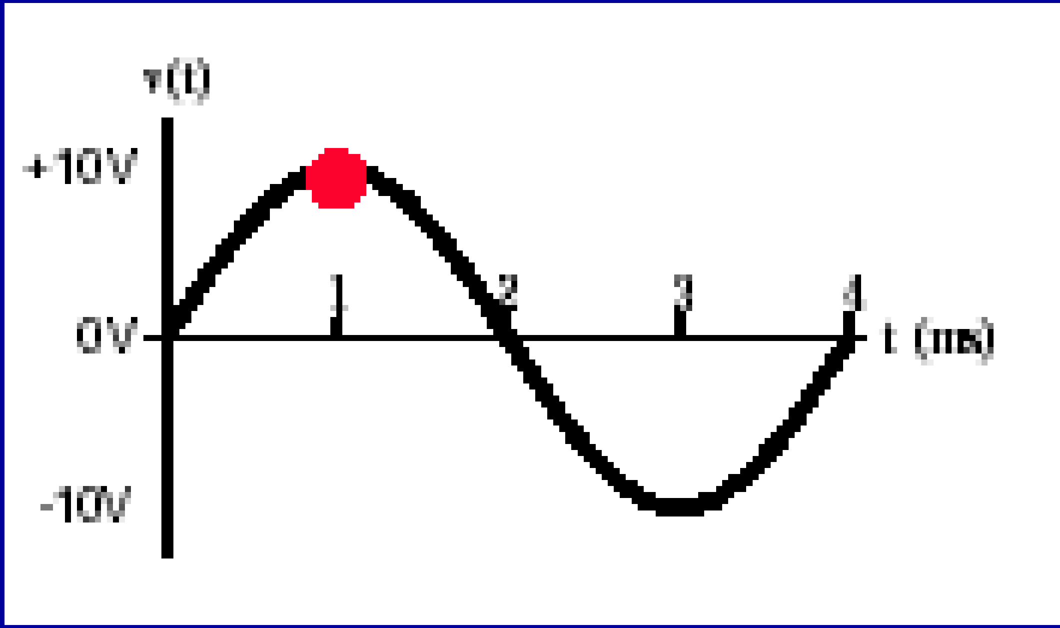


AC Wave – *value varies with time*

+10V

at

t* = 1 *ms

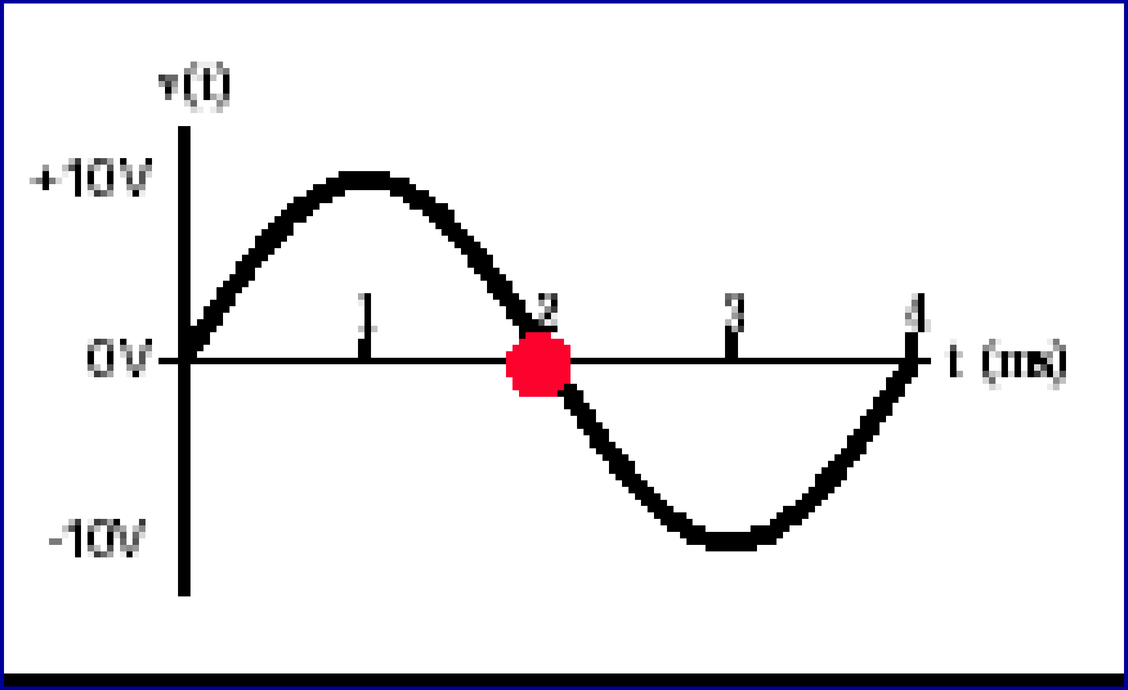


AC Wave – *value varies with time*

0V

at

t **= 2 *ms***

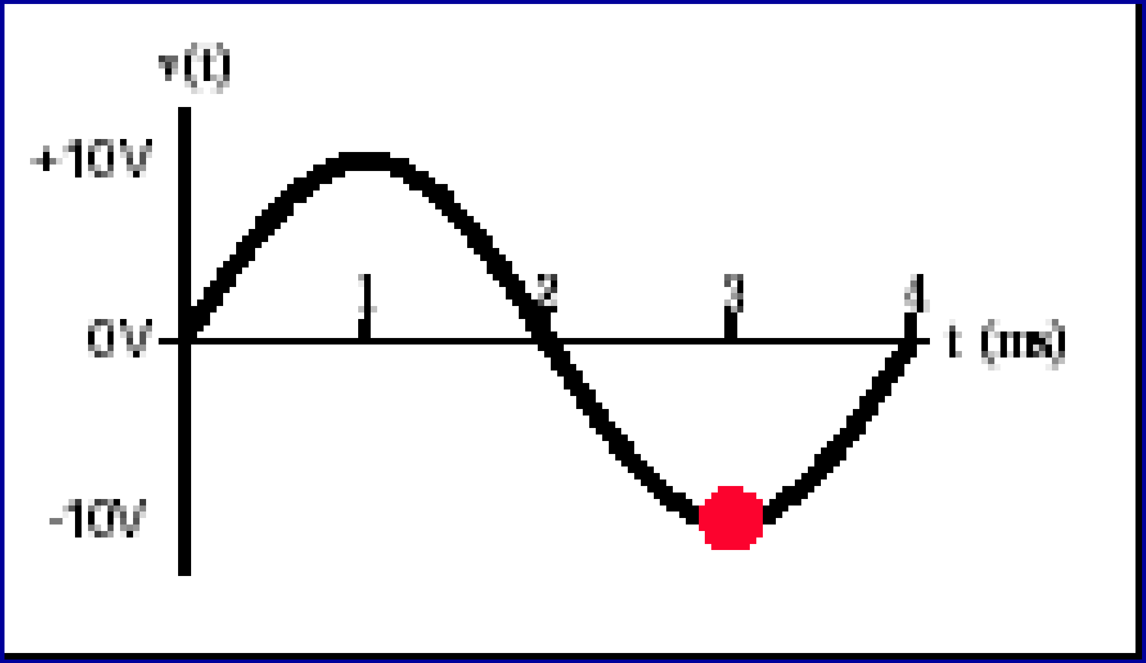


AC Wave – *value varies with time*

-10V

at

t* = 3 *ms

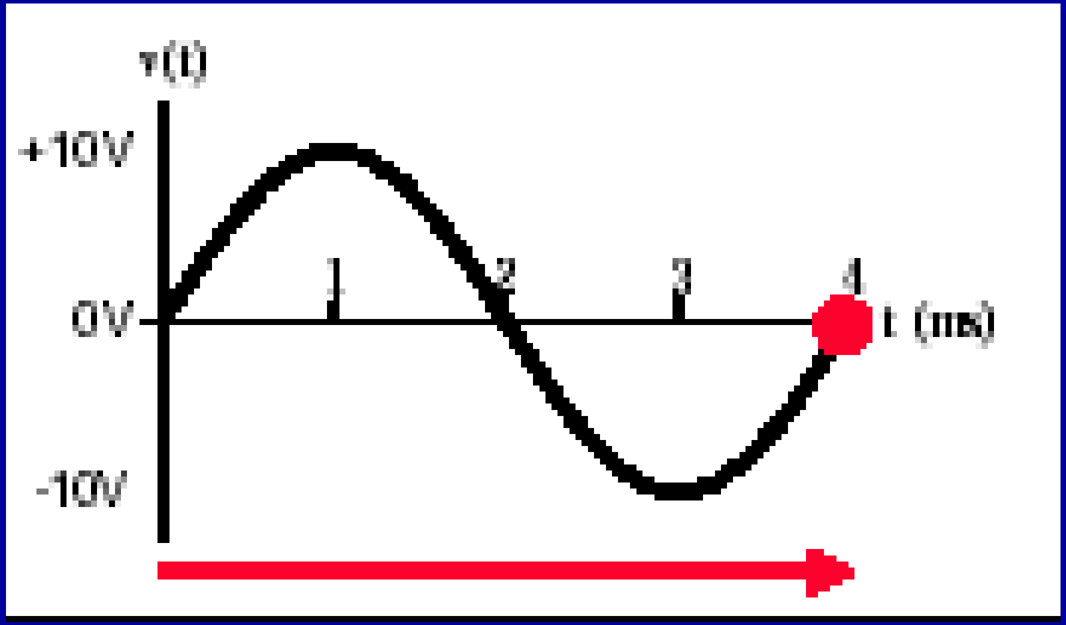


AC Wave – *value varies with time*

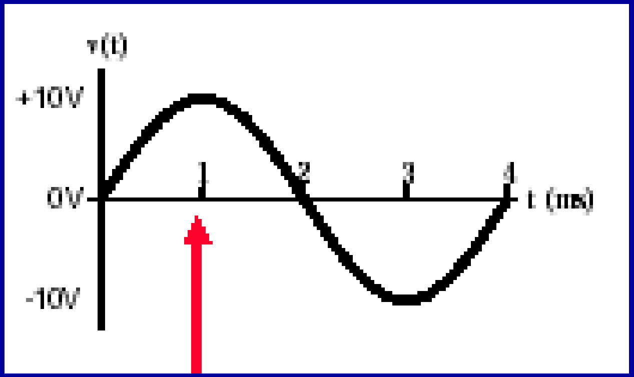
0V

at

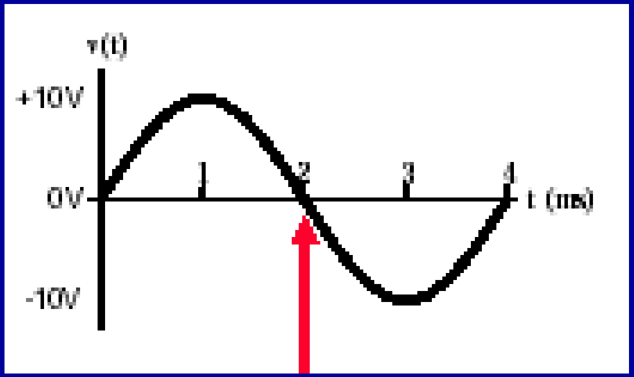
t* = 4 *ms



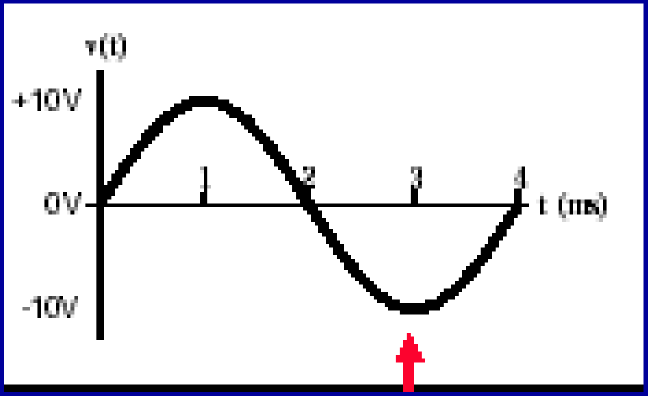
Time Axis



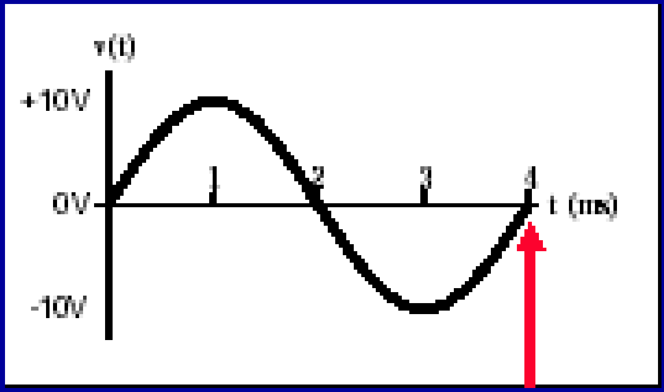
Quarter Cycle



Half Cycle

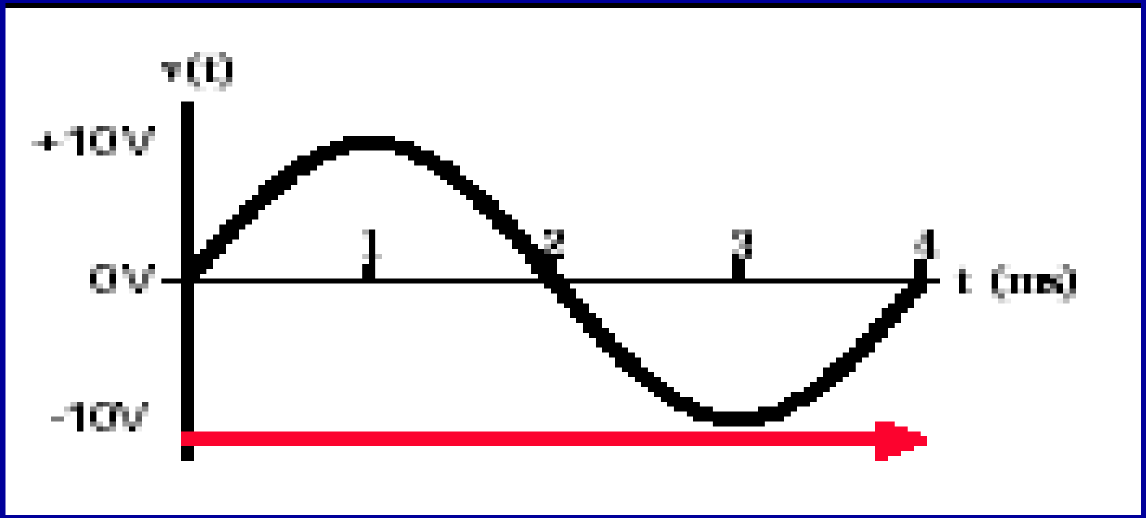


Three-quarter Cycle



Full Cycle

Key Concepts – *period*



Period – time for one cycle

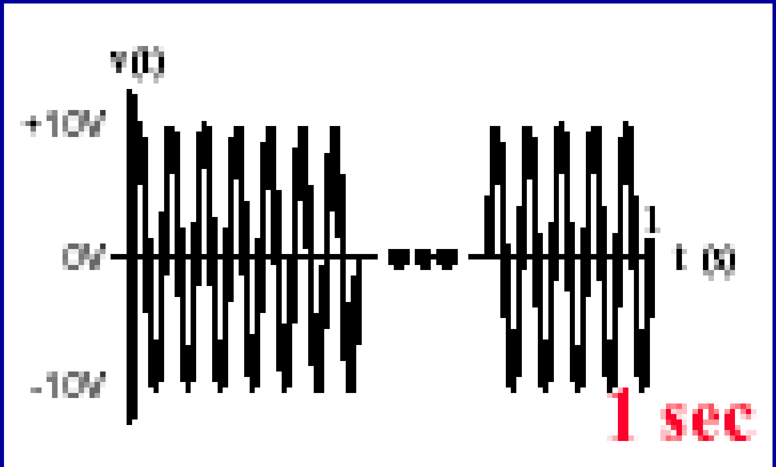
$$T = 4\ ms$$

Key Concepts – *frequency*

f **- frequency**

# of cycles in one second

$$f = 1/T$$



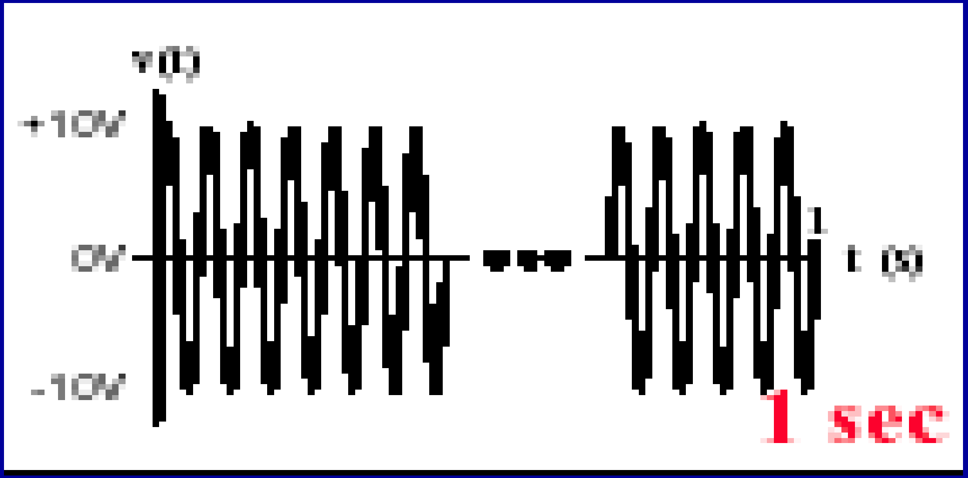
Where period ***T*** is time for one cycle

AC Wave – *frequency*

f – frequency

Units in Hertz (Hz)

Or cycles per second

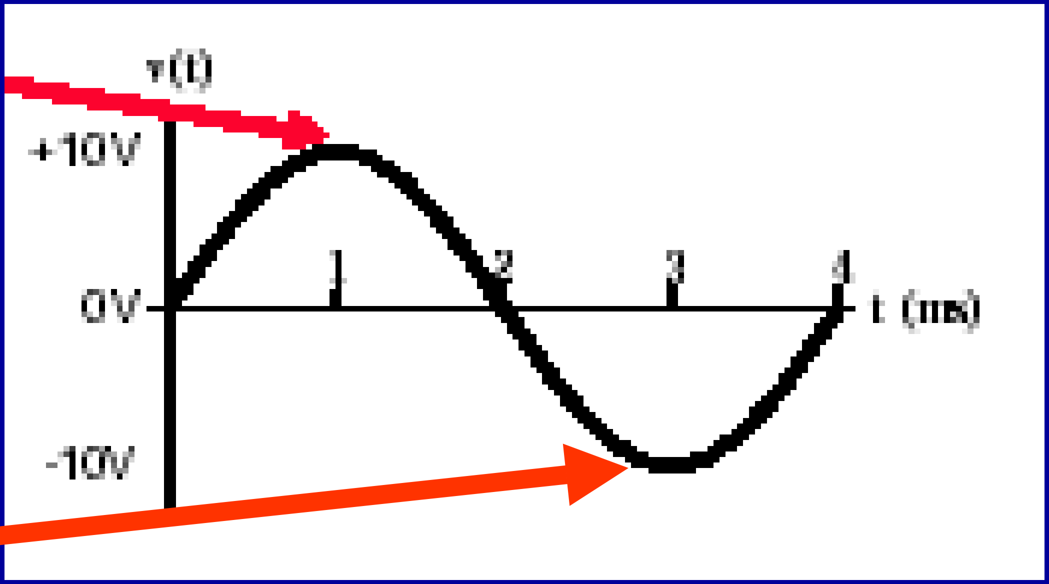


$$f = 1/T = ¼\ \mathrm{ms} = 250\ \mathrm{Hz}$$

AC Wave – *vertical axis*

Maximum or Peak

$$V_{max}=V_p=10V$$



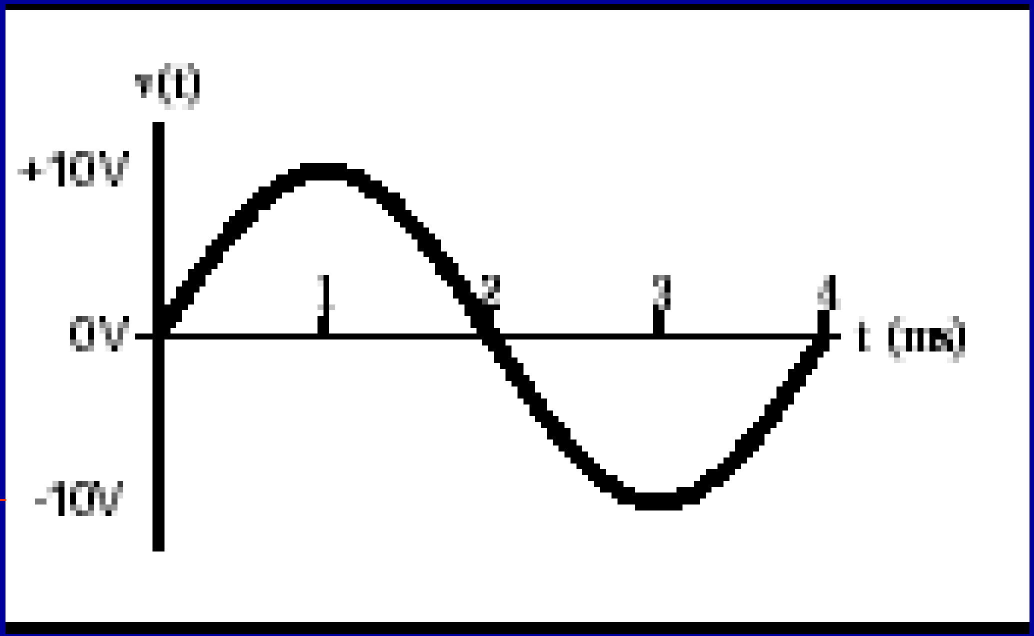
$$V_{min}= -10V$$

AC Wave – *vertical axis*

Peak

to

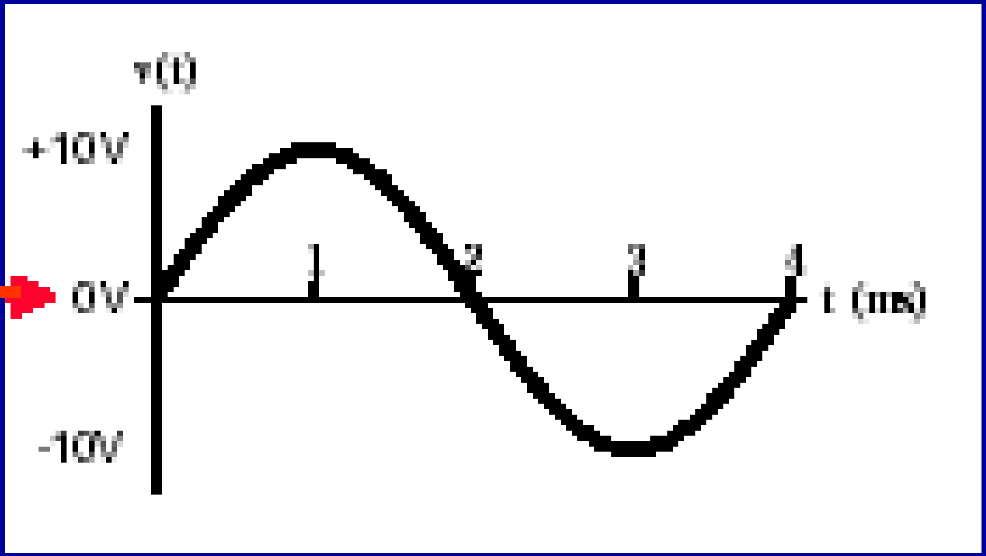
Peak



$$V_{pp} = V_{MAX} - V_{MIN} = 10\text{ V} - (-10\text{V}) = 20\text{ V}$$

AC Wave – *vertical axis*

Average or DC



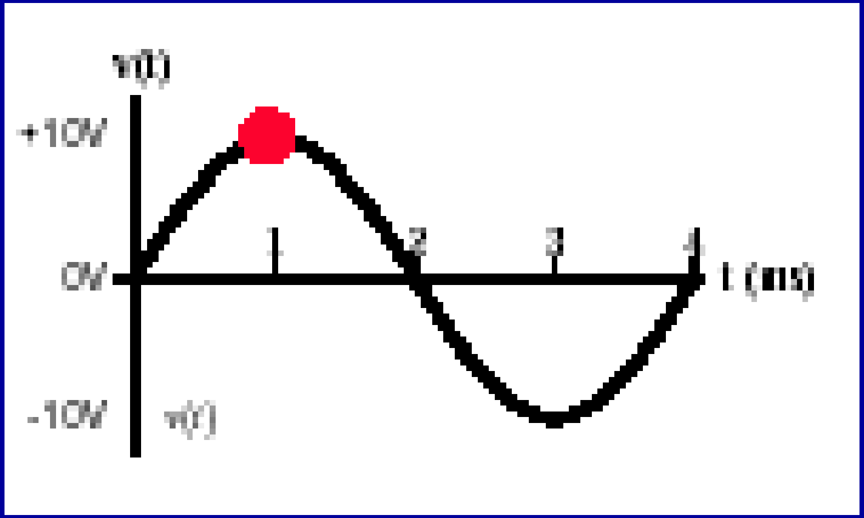
Pure AC signal

$$V_{AVG} = V_{DC} = 0$$

AC Wave – *sine wave*

$V_{max} = 10V$

$f = 250\ Hz$



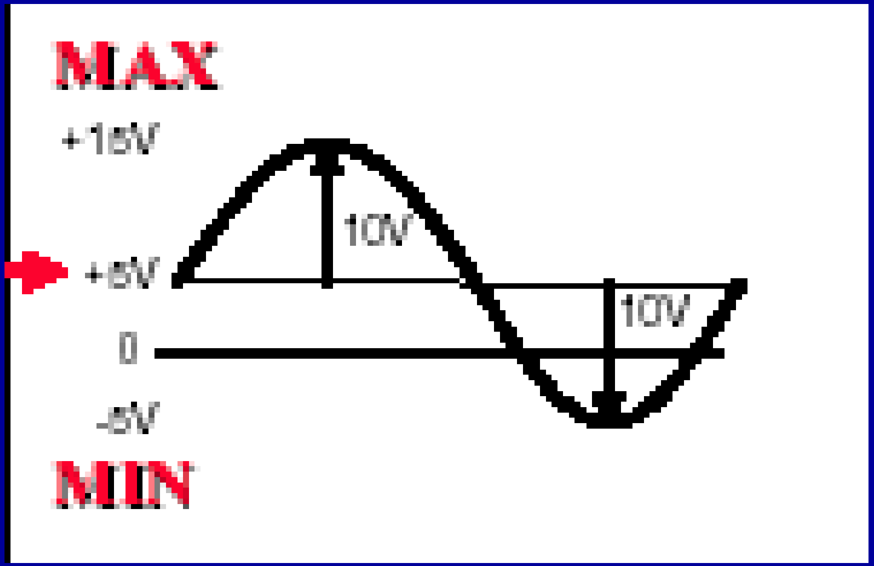
$$v(t) = V_m\, sin(2\pi f t) = V_m\, sin(\omega t)$$

$$v(t) = 10V\ sin(2 \cdot \pi \cdot 250\,Hz \cdot t)$$

$$v(0.001\ sec) = 10V\ sin(2 \cdot \pi \cdot 250\ Hz\ \cdot 0.001\ sec) = 10V$$

AC Wave – *with DC offset*

Average or DC



With 5V DC offset

$$V_{AVG} = V_{DC} = +5V$$

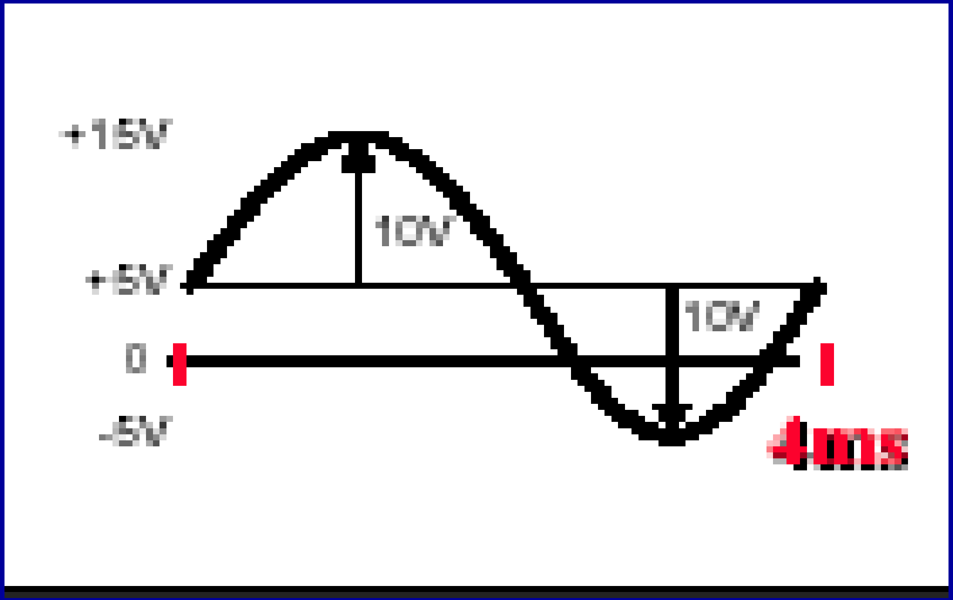
AC Wave – *sine wave*

V_{max}=10V

V_{DC}=+5V

T=4 ms

f= 250 Hz



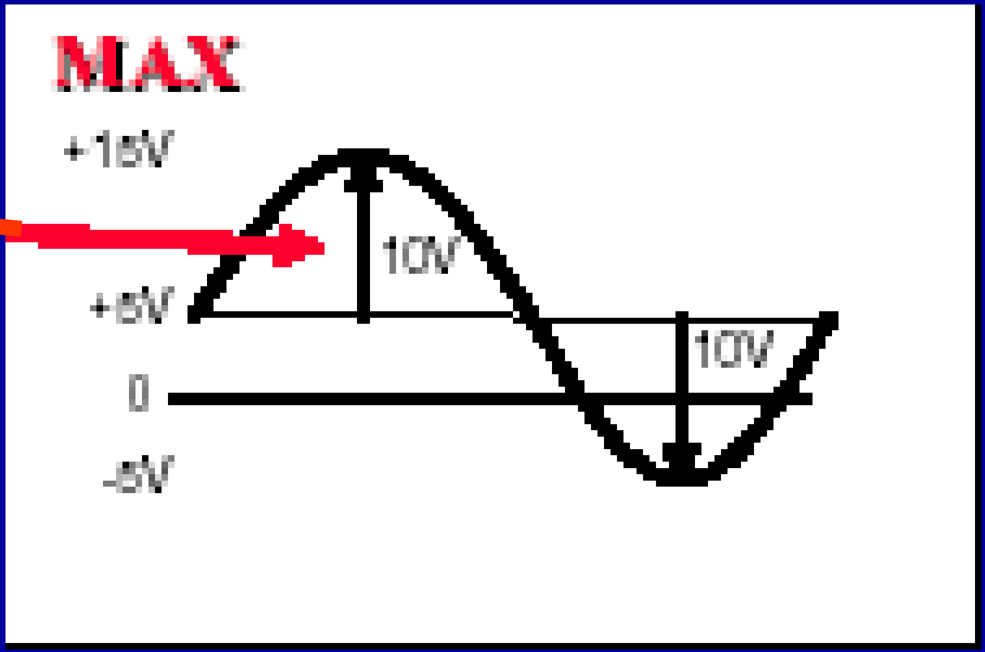
$$v(t) = V_{DC} + V_m \sin(2\pi f t)$$

$$v(t) = 5V + 10V \sin(2 \cdot \pi \cdot 250 Hz \cdot t)$$

AC Wave – *with DC offset*

V_m

Peak Amplitude



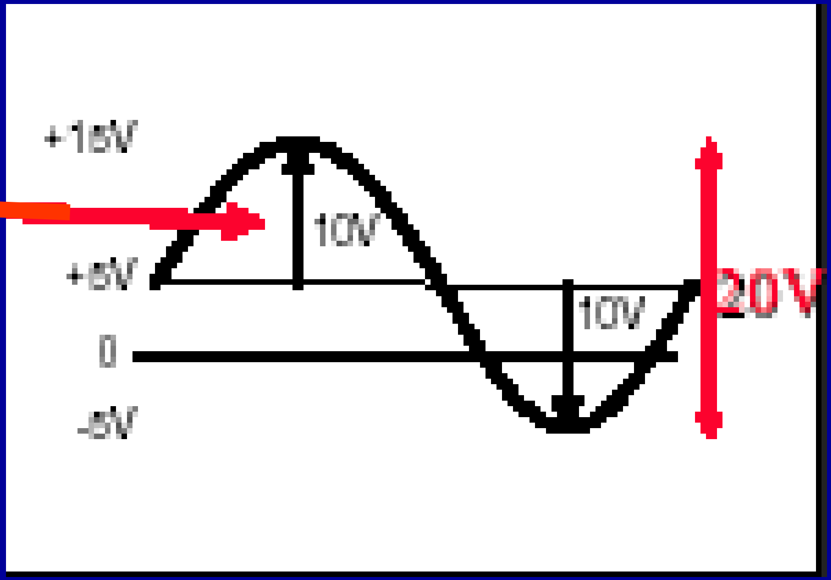
$$V_{max} = V_p = V_{DC} + V_m$$

$$= 5V + 10V = 15V$$

AC Wave – *with DC offset*

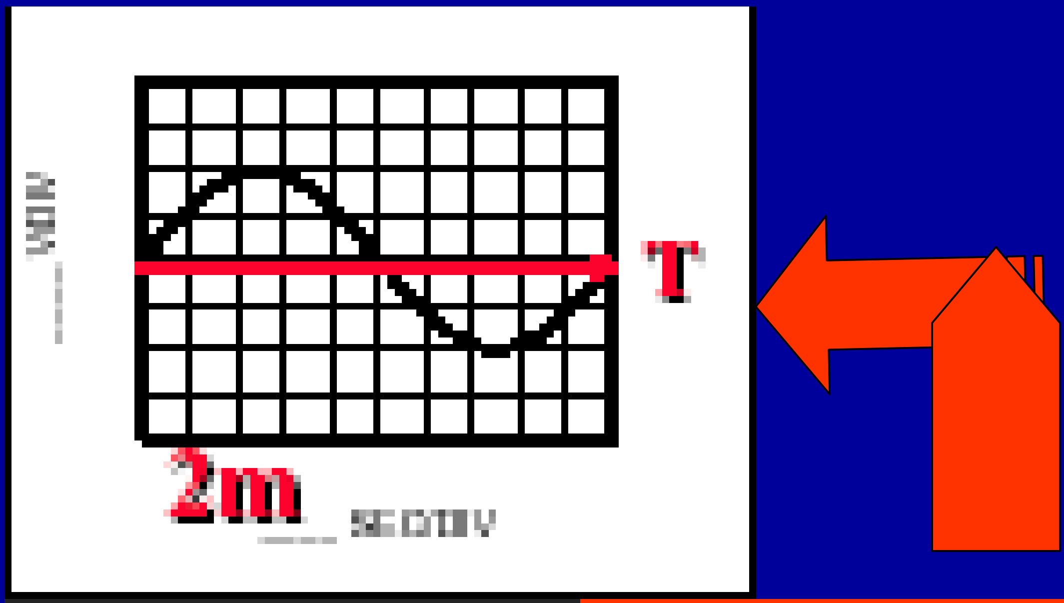
V_m

Peak Amplitude



$$V_m = \tfrac{1}{2}\,V_{pp} = \tfrac{1}{2}\,(20V) = 10V$$

Scope Display – *period measurement*



T = 10DIV x 2 ms/DIV = 20 ms

10 divisions

$f = 1/T$ =1/20 ms=50 Hz

Sweep Rate: 2ms/Div

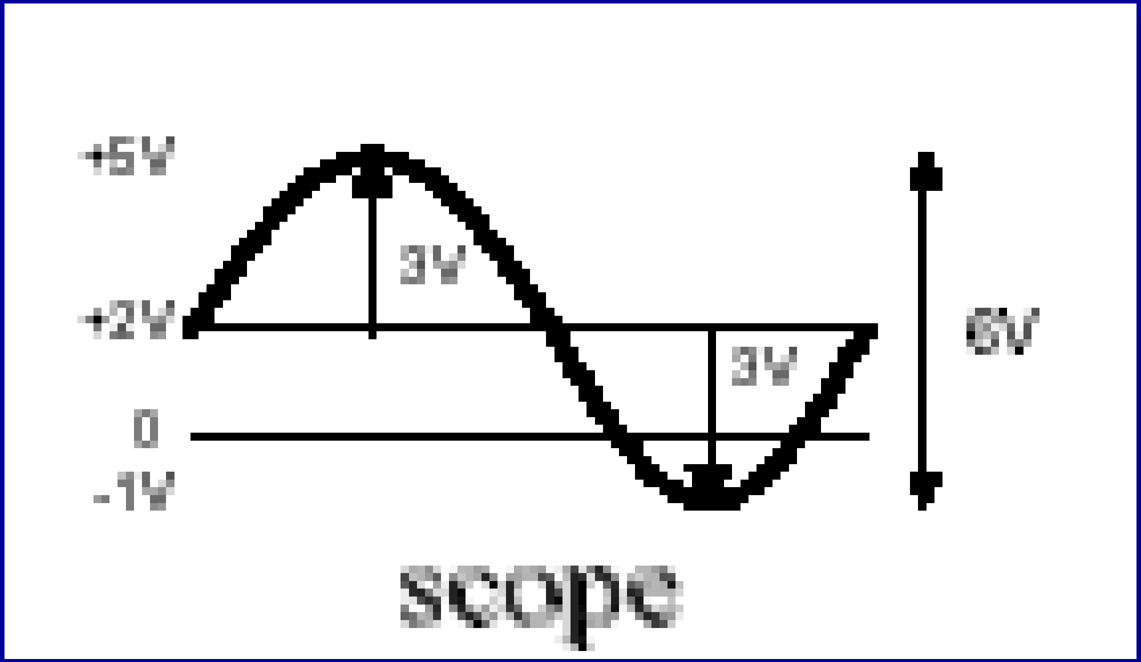
Average (DC)Value – *DMM*

DMM

DC mode

Voltage

+2.000 V

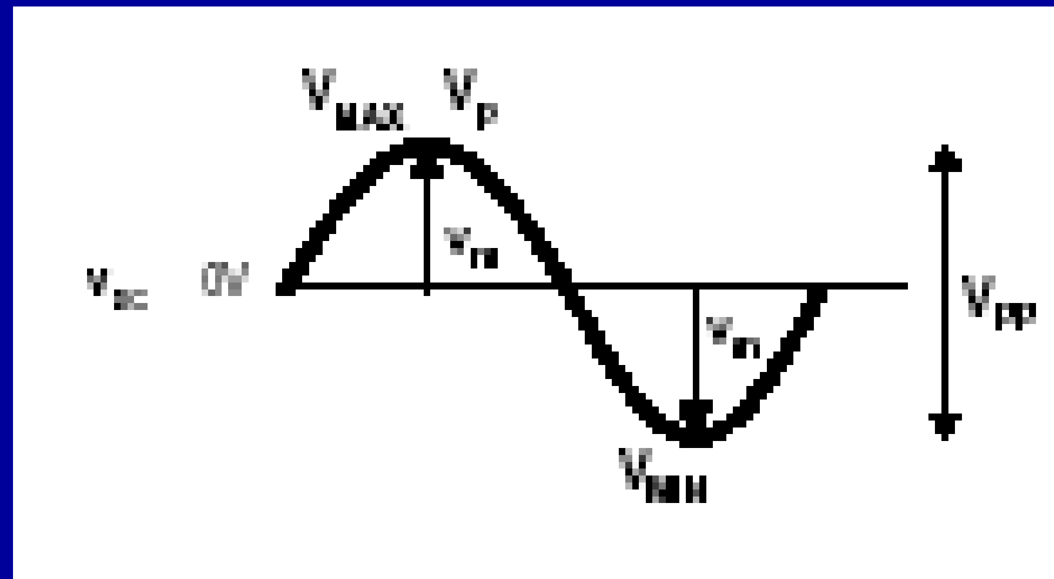


DMM: easy, accurate reading.

Effective (RMS)Value

Pure AC Signal

DC offset is 0V



DC is zero, but AC voltage exists

Need an AC measurement

Effective (RMS)Value

Root Mean Square

1. Square it
2. Take the mean
3. Take the square root

Effective (RMS)Value

Mathematically

From Calculus

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt}$$

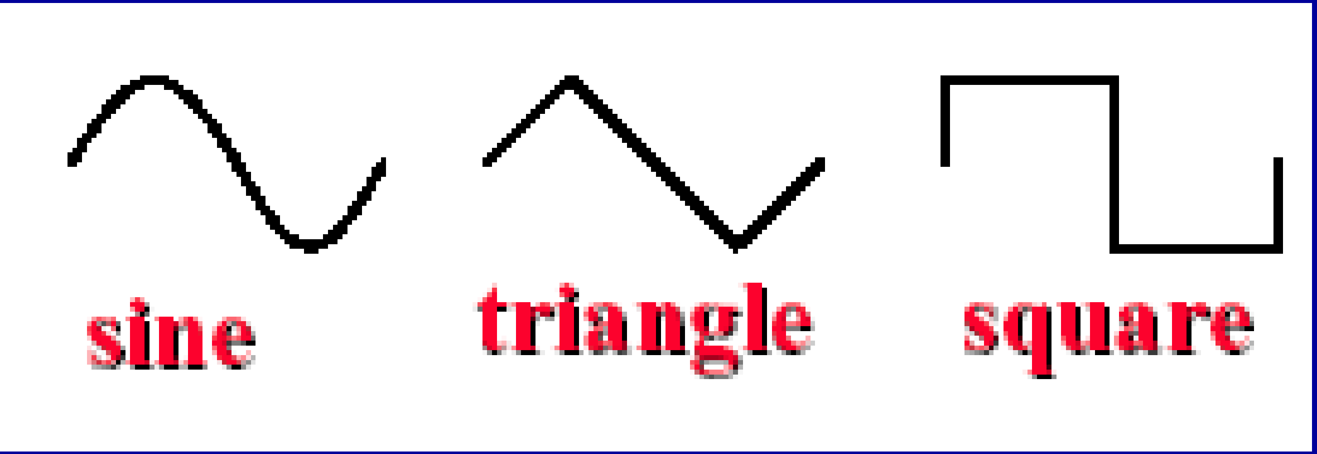
Square

Mean

Root

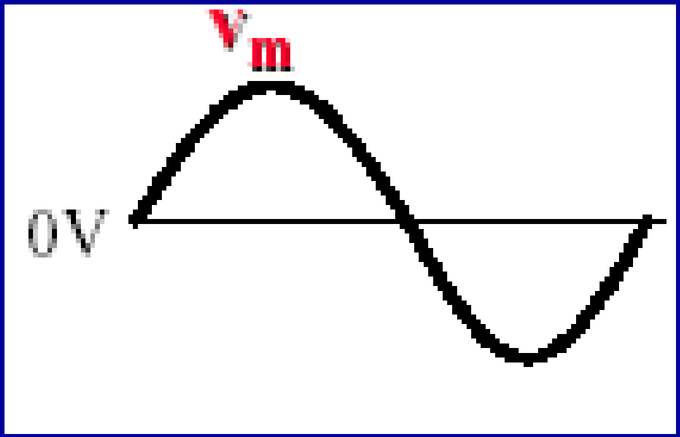
Effective (RMS)Value

Depends upon the shape!



Effective (RMS) Value

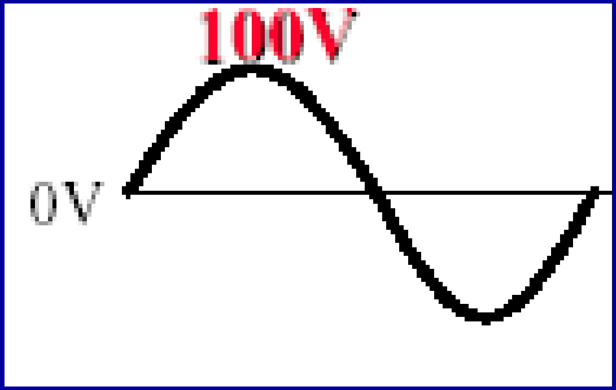
Sine



$$V_{RMS} = \frac{V_m}{\sqrt{2}}$$

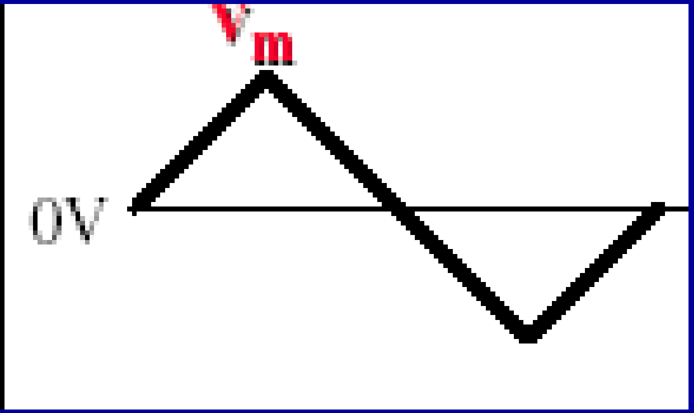
$$V_{RMS} = 0.707V_m$$

$$V_{RMS} = \frac{100V}{\sqrt{2}} = 70.7V$$



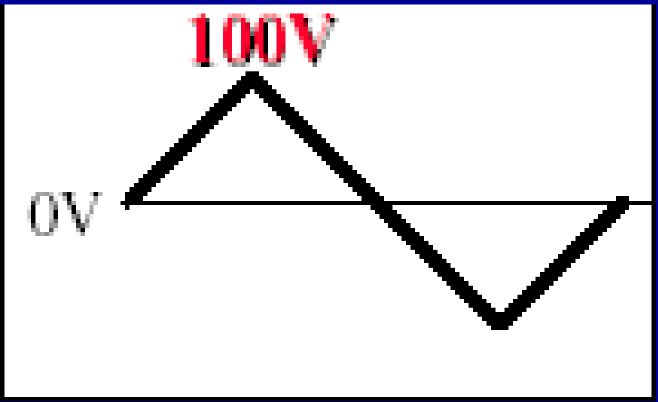
Effective (RMS)Value

Triangle

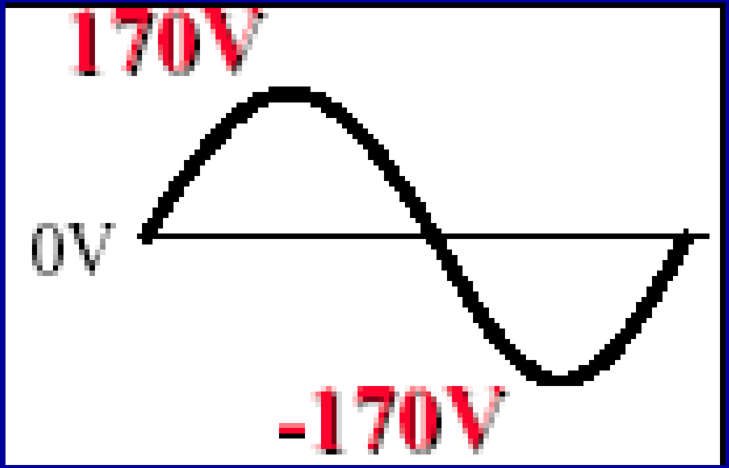


$$V_{RMS} = \frac{V_m}{\sqrt{3}}$$

$$V_{RMS} = \frac{100V}{\sqrt{3}} = 57.8V$$



Effective (RMS)Value



US commercial voltage

120V RMS

$120V_{RMS} = 170V_p = 340\ V_{pp}$

Frequency of 60 Hz

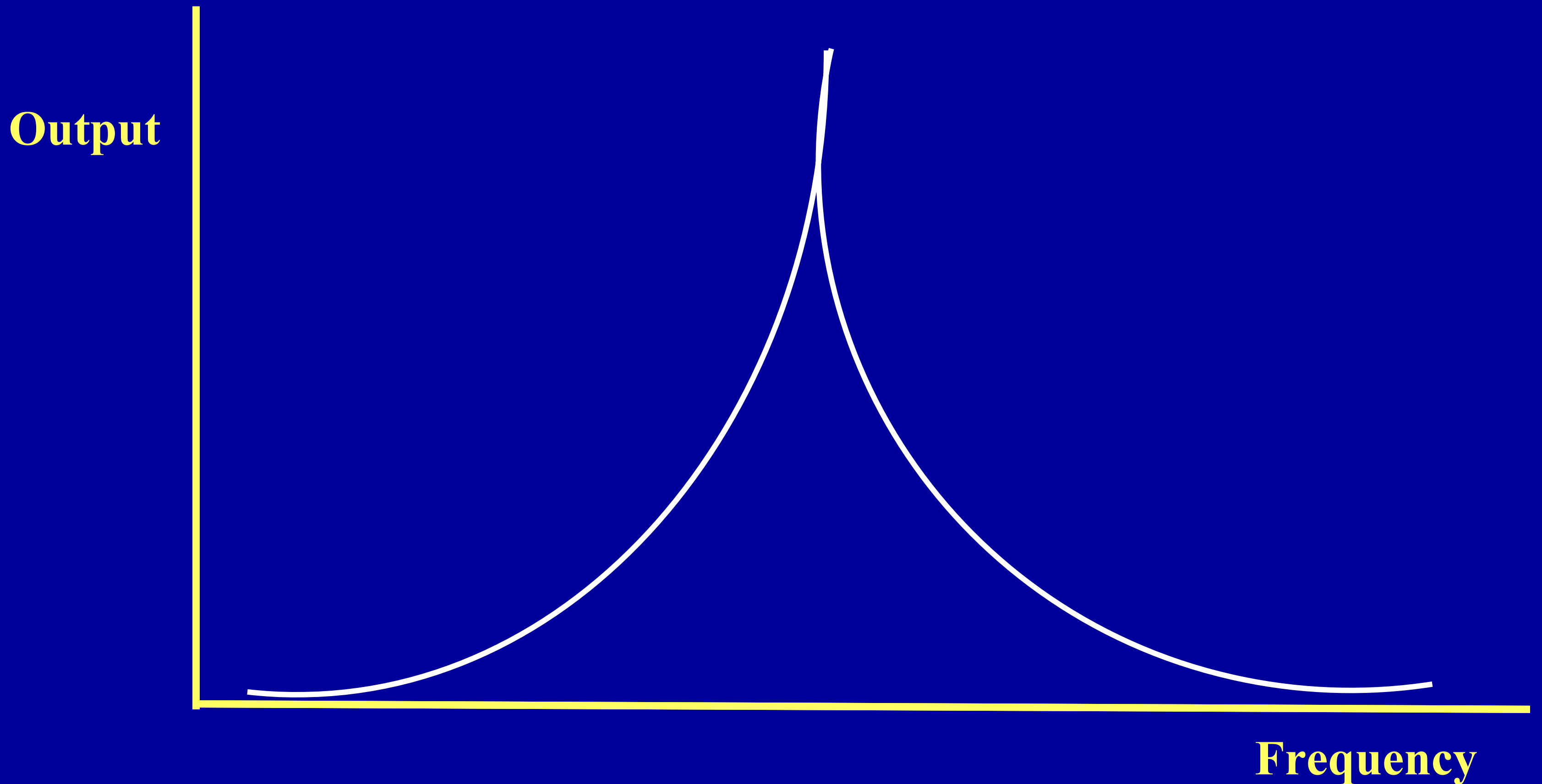
Period = 16.7 ms

Resonance

- Resonance Phenomena
- Schematic
- Frequency Effects
- Resonant Frequency
- Quality Factor
- Selectivity
- V_R, V_L, V_C

Resonance Phenomena

Output versus Frequency

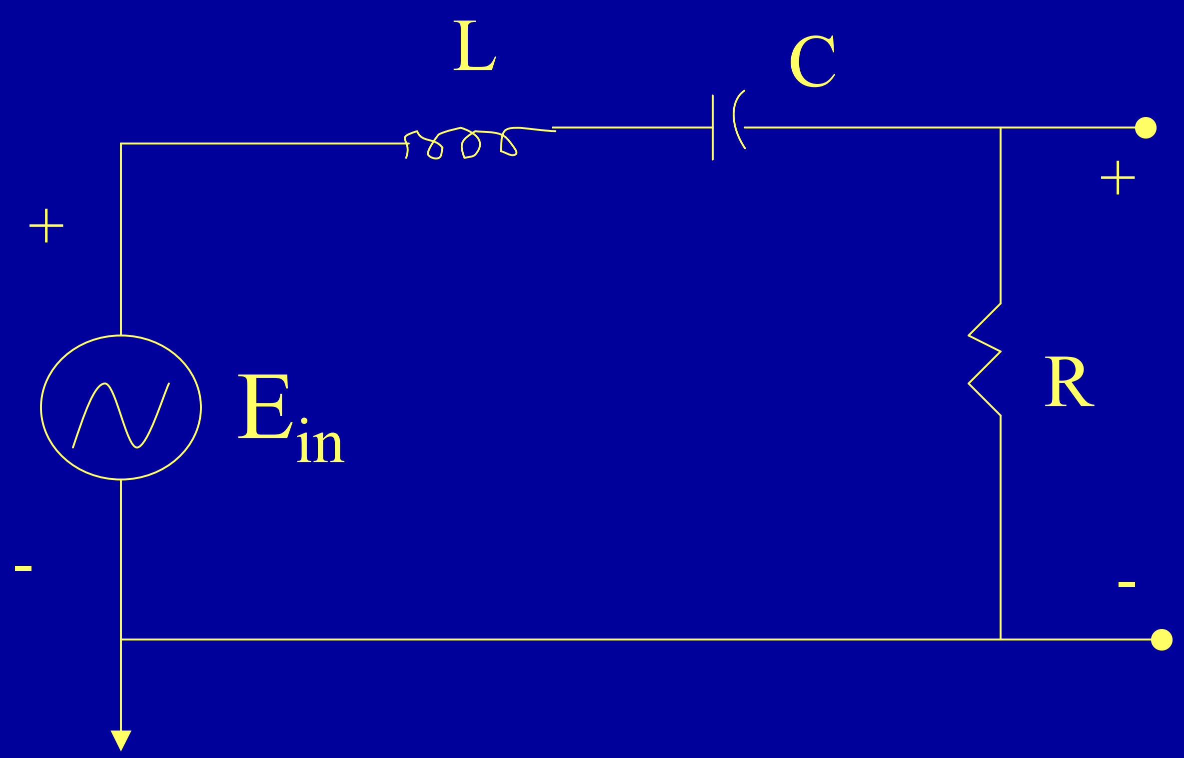


Resonance Phenomena

Requirements

- **Two energy storage devices**
- **180 degrees out of phase**
- **One releases energy while the other stores it**

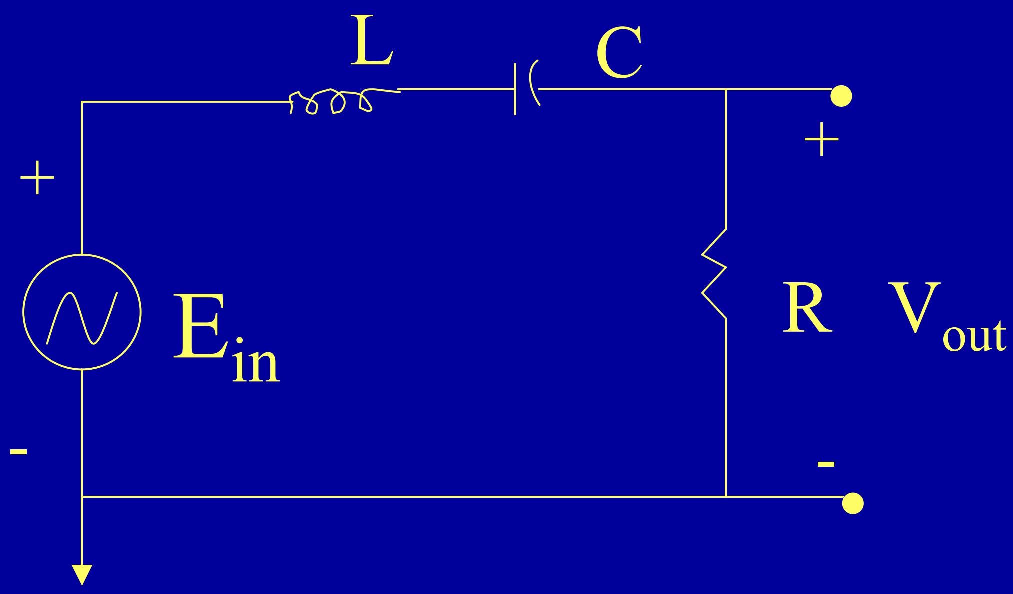
Series Resonant Circuit



Resonant Frequency

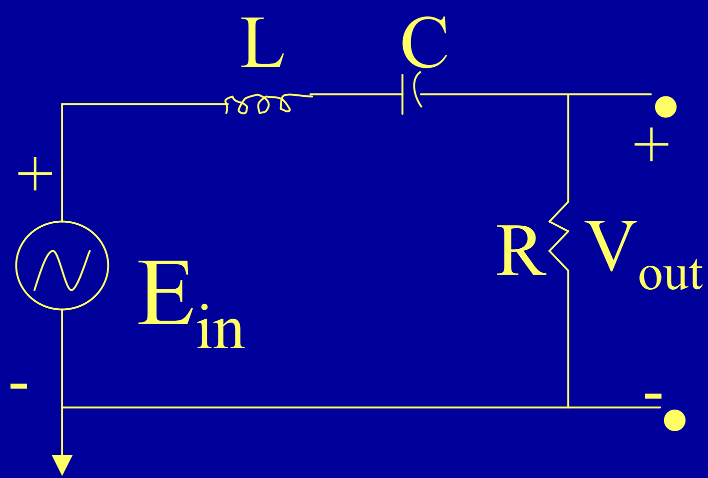
$$\overline{Z}_{total} = \overline{Z}_L + \overline{Z}_C + \overline{Z}_R$$

$$\overline{Z}_L = -\overline{Z}_C \quad X_L = X_C$$



$$\overline{Z}_{total} = \overline{Z}_R = R$$

Resonant Frequency: **Derivation**



$$X_L = X_C$$

$$2fL = \frac{1}{2fC}$$

$$f^2 = \frac{1}{4LC}$$

$$\boxed{f = \frac{1}{\sqrt{LC}}}$$

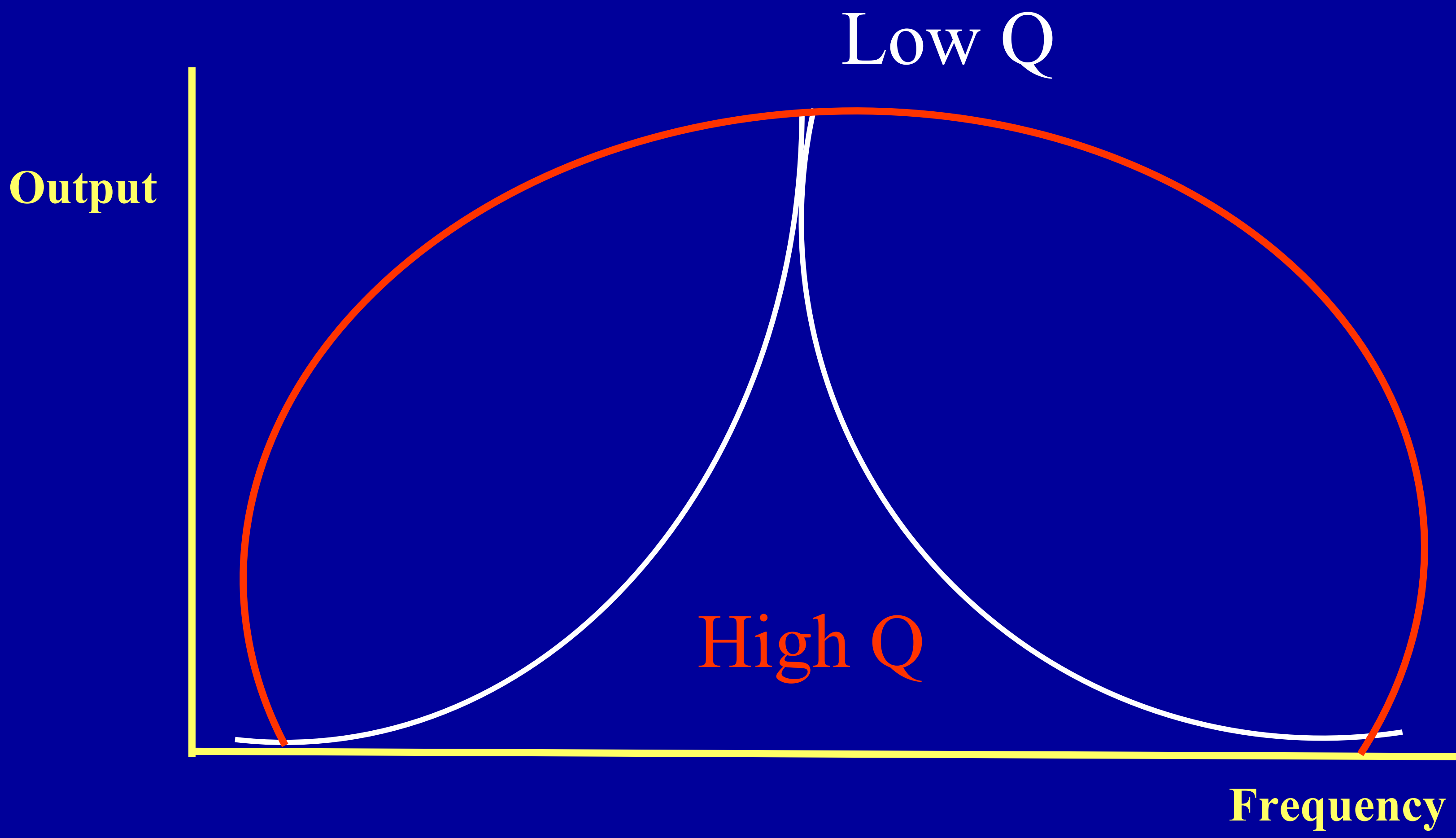
Quality Factor

$$Q = \frac{\text{reactive power}}{\text{resistive power}}$$

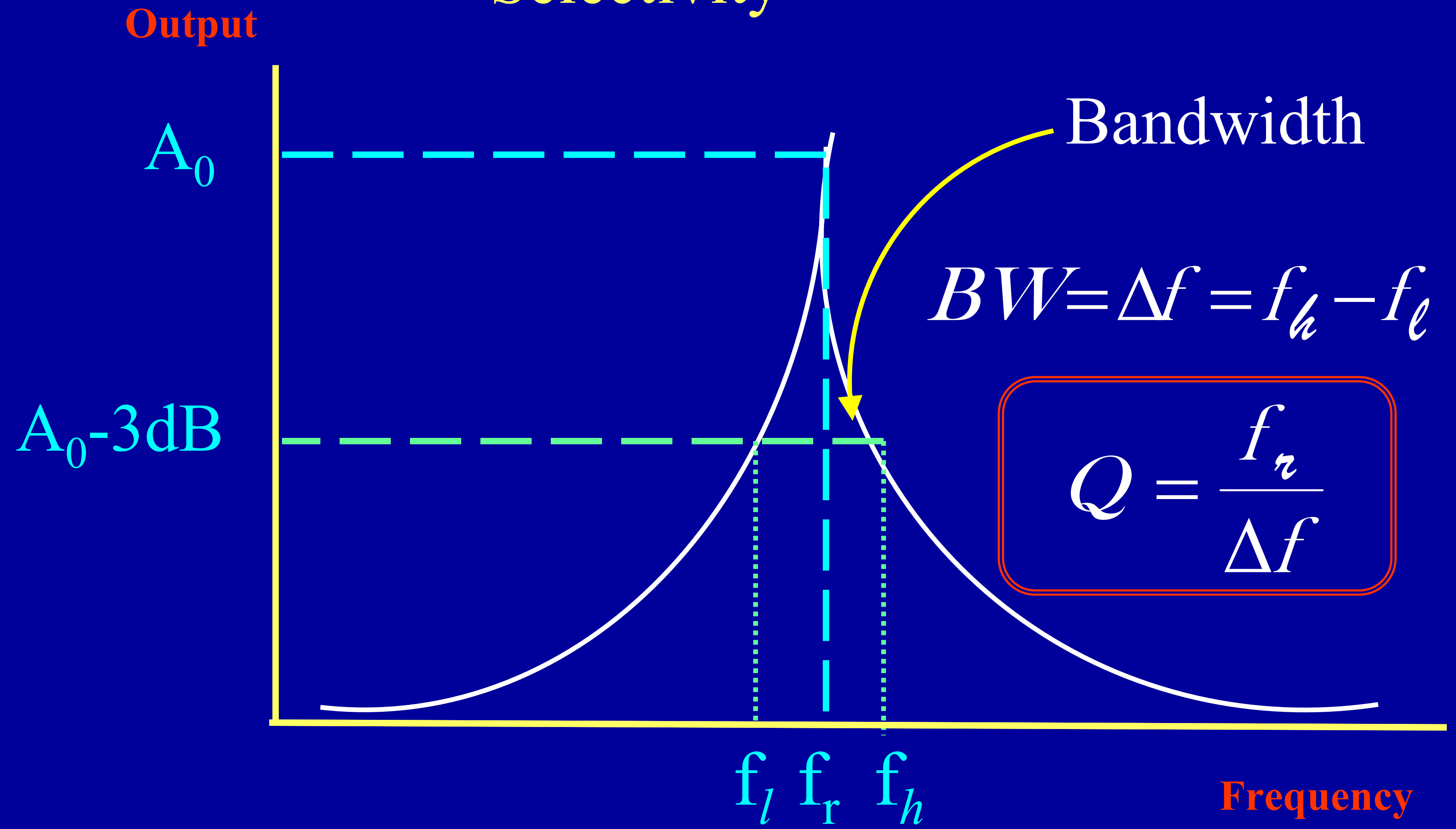
I is the same in a series circuit

$$Q = \frac{I^2 X}{I^2 R}$$

$$\boxed{Q = \frac{X}{R}}$$


Low Q
Output
High Q
Frequency

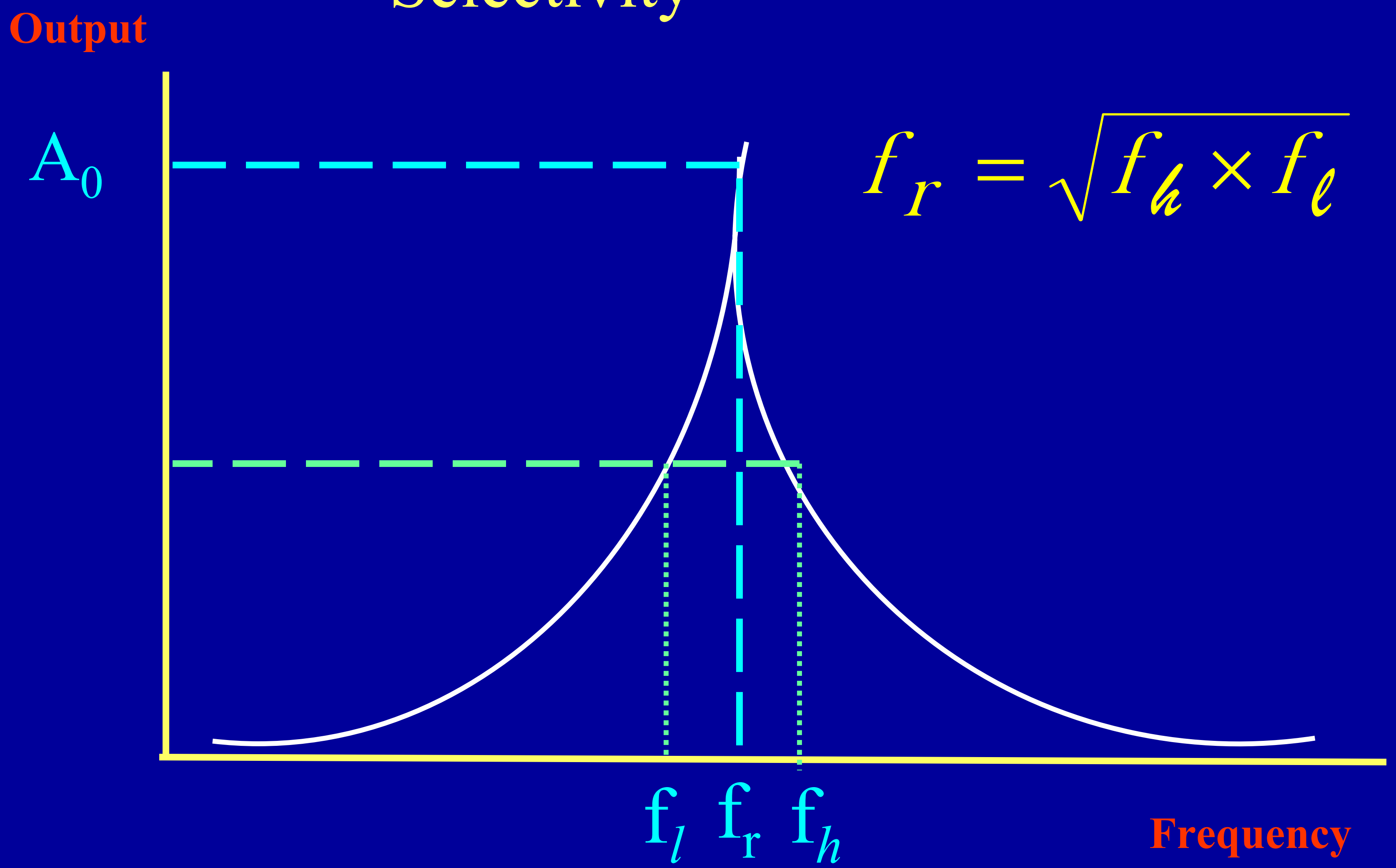
Selectivity



$$BW = \Delta f = f_h - f_\ell$$

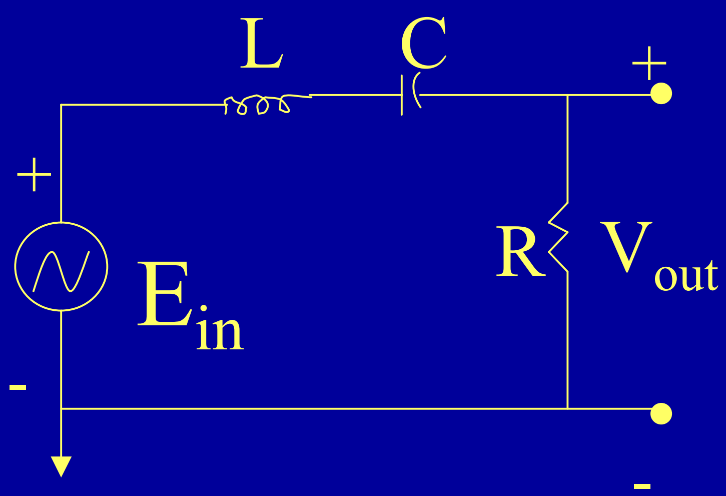
$$Q = \frac{f_r}{\Delta f}$$

Selectivity



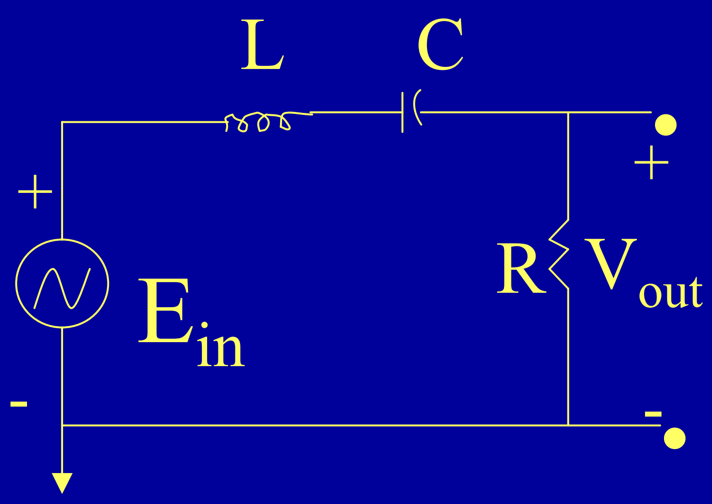
$$f_r = \sqrt{f_h \times f_\ell}$$

V_R, V_L, V_C Responses



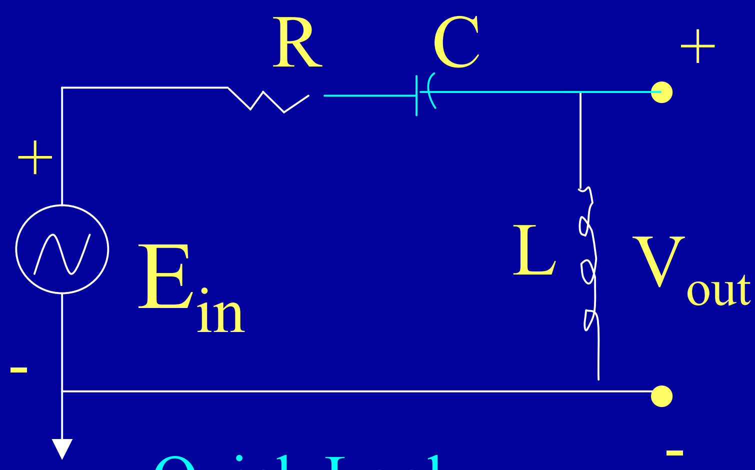
- At resonance I is ____?____
  - (min,ave,max)
- Max
- V_R, V_L, V_C are ___?___
  - (small,large)
- Large

V_R, V_L, V_C Responses



- At resonance
  - $V_L = V_C >> E_{in}$
- V_L is __?__
  - (LP,HP,BP)

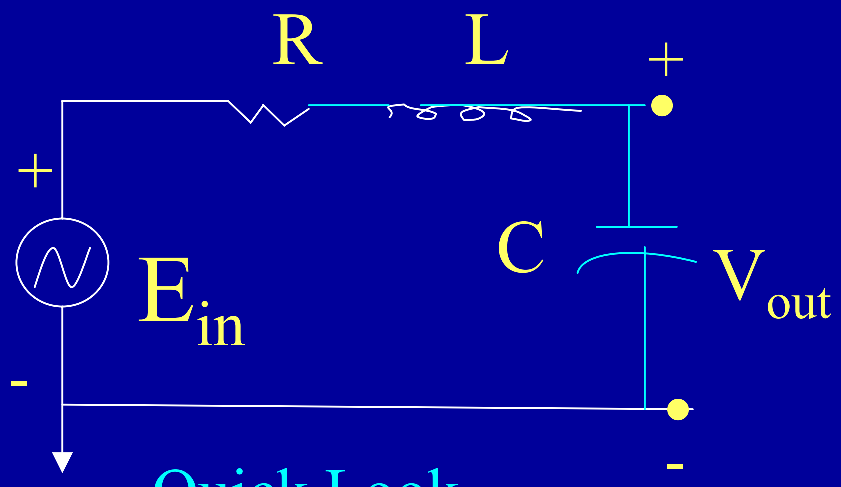
V_L is High Pass



Quick Look

$$\overline{G}_{f=0} = 0 \qquad \overline{G}_{f\to\infty} = 1$$

V_C is Low Pass



Quick Look

$$\overline{G}_{f=0} = 1 \qquad \overline{G}_{f\to\infty} = 0$$